Marco Zambon due Tu 6/10/2009

Metodos matematicos da fisica: Homework 1

1 (2 pt)**.** Let (V, Ω) be a symplectic vector space of dimension $2n$. Show that if $W \subset V$ is a linear subspace of dimension $2n-1$, then W is coisotropic.

2 (6 pt)**.** Let (V, Ω) be a symplectic vector space of dimension $2n$. Let $e_1, \dots, e_n, f_1, \dots, f_n$ be a basis of V and denote by $e_1^*, \cdots, e_n^*, f_1^*, \dots, f_n^*$ the dual basis of V^*. Suppose $\Omega(e_i, e_j) = 0 = \Omega(f_i, f_j), \Omega(e_i, f_j) = \delta_{ij}$.

i) Show that $\Omega = \sum_i e_i^* \wedge f_i^*$.

ii) Show that $\Omega^n := \Omega \wedge \cdots \wedge \Omega$ is a non-zero element of $\wedge^{2n} V^* \cong \mathbb{R}$.

3 (4 pt)**.** Consider the vector field $X = \sum x_i \frac{\partial}{\partial x_i}$ on $\mathbb{R}^n$. Write down a formula for the flow Φ_t of X.

4 (8 pt)**.** Consider $\mathbb{R}^4$ with coordinates x, y, w, z and the 2-form $\alpha = a dx \wedge dy$ where $a \in C^\infty(\mathbb{R}^4)$.

i) At each $p \in \mathbb{R}^4$, describe $ker(\alpha_p) := \{v \in T_p\mathbb{R}^4 : \alpha_p(v, w) = 0 \ \ \forall w \in T_p\mathbb{R}^4\}$

ii) Describe, in terms on a, when $d\alpha = 0$.

iii) Show that if $d\alpha = 0$ then there exists $\beta \in \Omega^1(\mathbb{R}^4)$ with $d\beta = \alpha$.

iv) Show that if $d\alpha \neq 0$ then there can *not* exist $\beta \in \Omega^1(\mathbb{R}^4)$ with $d\beta = \alpha$.